A NOTE ON RAMANUJAN'S GENERAL THETA FUNCTION

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Abstract: In this paper, Ramanujan's general theta function has been generalized and its properties have been discussed.

Keywords and Phrases: Jacobi's theta functions, identities, Ramanujan's general theta function, generalized Ramanujan's theta function.

2010 Mathematics Subject Classification: 33D15, 11P81, 11P82, 11F11, 05A17, 05A30.

1. Introduction

Jacobi in 1829 [3] defined following four functions which are called Jacobi's theta functions,

$$\theta_1(z,q) = 2\sum_{n=0}^{\infty}(-1)^n q^{\left(n+\frac{1}{2}\right)^2}\sin(2n+1)z, \tag{1.1}$$

$$\theta_2(z,q) = 2\sum_{n=0}^{\infty} q^{\left(n+\frac{1}{2}\right)^2}\cos(2n+1)z, \tag{1.2}$$

$$\theta_3(z,q) = 1 + 2\sum_{n=1}^{\infty} q^{n^2}\cos 2nz \tag{1.3}$$

and

$$\theta_4(z,q) = 1 + 2\sum_{n=1}^{\infty}(-1)^n q^{n^2}\cos 2nz. \tag{1.4}$$